

# Quantum reference systems: reconciling locality with quantum mechanics

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**Abstract.** The status of locality in quantum mechanics is analyzed from a nonstandard point of view. It is assumed that quantum states are relative in the sense that they depend on and are defined with respect to some bigger physical system which contains the former system as a subsystem. Hence, the bigger system acts as a reference system. It is shown that quantum mechanics can be reformulated in accordance with this new physical assumption.

Additional laws express the (probabilistic) relation among states which refer to different quantum reference systems. They replace von Neumann's postulate about the measurement (collapse of the wave function). The dependence of the quantum states on the quantum reference systems resolves the apparent contradiction connected with the measurement (Schrödinger's cat paradox). There is another important consequence of this dependence: states may not be comparable, i.e., they cannot be checked by suitable measurements simultaneously. This special circumstance is fully reflected mathematically by the theory. Especially, it is shown that certain joint probabilities (or the corresponding combined events) which play a vital role in any proof of Bell's theorem do not exist. The conclusion is that the principle of locality holds true in quantum mechanics, and one has to give up instead of locality an intuitively natural-looking feature of realism, namely, the comparability of existing states.

# 1 Introduction

During the development of physics research is extended to phenomena at such spatial, time or energy scales that are far beyond the range of everyday experience. These phenomena sometimes force us to revise our previous concepts which have seemed to us natural or even indispensable, but are actually rooted in our limited previous experience and finally prove to be of approximate validity. This is reflected in the growing level of abstraction of physical theories. Quantum phenomena have forced already several such revisions. A primary example is the surrender of determinism, but the wavelike behaviour of particles, the existence of discrete energy levels and Heisenberg's uncertainty relations also imply substantial revisions of some basic features of classical physics. It is well known that quantum phenomena force us to surrender or revise at least one more very basic concept: Bell's theorem tells us that that we have to give up (or revise) either locality, or realism, or inductive inference[1]-[9]. On the other hand, these concepts are so deeply rooted in scientific thinking that one is reluctant to give up any of them. Many people think that perhaps locality is the weakest concept of the three (cf. arguments for nonlocality in Ref.[10] and counterarguments in Ref.[11]). Nevertheless, all the fundamental equations of physics satisfy the principle of locality, including e.g. the standard model of the elementary particles, so it is rather implausible that locality would be violated just in quantum measurements. The myth of nonlocality is much less attractive today than it has been in the fifties or sixties when people tried to build nonlocal quantum field theories.

There are at least two other myths related to quantum mechanics: one of them is the notorious belief that around some large particle number or mass quantum mechanics gradually becomes invalid and classical mechanics starts to be correct. Actually this very simple view has no experimental support at all. On the contrary, all the available experimental results confirm the validity of quantum mechanics, even in the macroscopic situation of superfluid He. The actual motivation of the above view is the paradox of quantum measurements, which involves the third myth, the fictitious collapse of the wave function[12]. This process was

invented for the interpretation of quantum mechanics, i.e., for making contact between theoretical results and the experience. Nevertheless, if considered an actual physical process, it contradicts the Schrödinger equation and violates the principle of locality.

In the present paper we show that a consistent theory can be constructed without accepting any of the three myths above. As a guiding principle, we try to confine the number of assumptions to a minimum and base our considerations on logical clarity and consistency. Obviously, this is all we can safely do in a range of phenomena where our intuition does not work. As expected, one has to revise rather substantially some basic concepts. Namely, it will be shown that one has to give up the belief that a physical system at a given instant of time has a quantum state in an absolute sense, i.e., a state which depends only on the system to be described (apart from unitary transformations which may appear if another coordinate system is chosen). Instead, quantum states will express basically a relation between a system to be described and another system containing the former one. The latter, bigger system acts as reference system[13]. This new kind of dependence on reference systems will be explained in detail in Section 2. The idea that quantum states are relational has been put forward (in mathematically and conceptually different ways) in Refs.[14],[15]. The relational nature of quantum mechanics is expressed also by the proposal that correlations (rather than states) are the basic entities ("correlations without correlata") [16].

In Section 3. the basic rules of quantum mechanics are formulated in a way that takes into account the dependence of the states on quantum reference systems. von Neumann's measurement postulate will be replaced by the rules which express the indeterministic relation among states defined with respect to different quantum reference systems. The most important of these rules, which has no counterpart in standard quantum mechanics, utilizes the eigenstates of the reduced density matrix. These eigenstates (Schmidt states[17]) play a central role also in the modal interpretations<sup>1</sup>[18]. Their significance has also been emphasized within the framework of decoherence theory [19], [20]. According to the present

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<sup>1</sup>One may even consider the present approach a modal interpretation in the sense that it also aims at giving a physical meaning to the quantum formalism ("properties attribution") like modal interpretations.

approach, Schrödinger's equation is universally valid and measurements are considered as any other interaction between two physical systems. If the dynamics of this interaction has the special form of a perfect (von Neumann type) measurement, the usual quantum mechanical expressions for the probabilities are recovered.

The dependence of quantum states on quantum reference systems (as defined in the present paper) leads to an even more radical consequence: some states which exist may not be compared. This is totally unusual for our intuition and classical experience, but if the states which we would like to compare are defined with respect to different reference systems, there is no logical necessity that one can indeed compare them. This basically influences the picture we have about reality. We cannot think of existing states as of letters in a big book. If two letters exist, we may compare them in principle, without changing them or their relationship. Thus (if we do not possess complete knowledge about them) it makes sense to speak about the joint probability of their simultaneous presence. In case of existing quantum states one may check any of them by a suitable measurement, without disturbing that particular state. But such a measurement usually does disturb the other states, therefore, these measurements usually cannot be performed simultaneously (or one by one) without influencing any of the relevant states. Thus one cannot give any operational definition of the joint probability of the simultaneous presence of states. This seems to be rather odd, and one might think that independently of this situation such probabilities must exist on the ground that each existing state somehow corresponds to an element of the reality. On the other hand, one should keep in mind that this set-like picture of the reality is based on the fact that in classical physics the property of comparability is always given. In the absence of that one has no reason to expect that the consequences remain intact. Let us emphasize that this modification of the concept of reality is completely independent of any influence of consciousness. Therefore, the resulting reality concept still preserves the idea that the world is made up of objects whose existence is independent of human consciousness. The non-comparability of states will be discussed in Section 4.

In Section 5. the derivation of Bell's inequality is analyzed from the point of view of the present modified quantum mechanics. It will be shown that the role of the "hidden variable" is played by a particular quantum state. It turns out that the usual "shadowing property" which would express local realism, does not follow, because the corresponding three-fold joint probability is physically meaningless due to the non-comparability of the relevant quantum states. On the other hand, all reasonable requirements of locality are fulfilled.

In Section 6. we summarize the results and conclude.

## 2 The basic new concept: quantum reference systems

Let us consider a simple example, namely, an idealized measurement of an  $\hat{S}_z$  spin component of a spin- $\frac{1}{2}$  particle. Be the particle  $P$  initially in the state

$$\alpha|\uparrow\rangle + \beta|\downarrow\rangle, \quad (1)$$

where  $|\alpha|^2 + |\beta|^2 = 1$  and the states  $|\uparrow\rangle$  and  $|\downarrow\rangle$  are the eigenstates of  $\hat{S}_z$  corresponding to the eigenvalues  $\frac{\hbar}{2}$  and  $-\frac{\hbar}{2}$ , respectively. The other quantum numbers and variables have been suppressed. The dynamics of the measurement is given by the relations  $|\uparrow\rangle |m_0\rangle \rightarrow |\uparrow\rangle |m_\uparrow\rangle$  and  $|\downarrow\rangle |m_0\rangle \rightarrow |\downarrow\rangle |m_\downarrow\rangle$ , where  $|m_0\rangle$  stands for the state of the measuring device  $M$  (e.g. a Stern-Gerlach apparatus) before the measurement (no spot on the photographic plate), while  $|m_\uparrow\rangle$  ( $|m_\downarrow\rangle$ ) is the state of the measuring device after the measurement that corresponds to the measured spin value  $\frac{\hbar}{2}$  ( $-\frac{\hbar}{2}$ ). The shorthand notation  $\rightarrow$  stands for the unitary time evolution during the measurement, which is assumed to fulfill the time dependent Schrödinger equation corresponding to the total Hamiltonian of the combined  $P+M$  system. As the initial state of the particle is given by Eq.(1), the linearity of the Schrödinger equation implies that the measurement process can be written as

$$(\alpha|\uparrow\rangle + \beta|\downarrow\rangle)|m_0\rangle \rightarrow |\Psi\rangle = \alpha|\uparrow\rangle |m_\uparrow\rangle + \beta|\downarrow\rangle |m_\downarrow\rangle. \quad (2)$$

This simplified dynamics is called a von Neumann type perfect measurement. Let us consider now the state of the measuring device  $M$  after the measurement. As the combined system

$P + M$  is in an entangled state, the measuring device has no own wave function and may be described by the *reduced density matrix*[21]

$$\hat{\rho}_M = Tr_P (|\Psi\rangle\langle\Psi|) = |m_\uparrow\rangle\langle m_\uparrow| |\alpha|^2 + |m_\downarrow\rangle\langle m_\downarrow| |\beta|^2 \quad , \quad (3)$$

where  $Tr_P$  stands for the trace operation in the Hilbert space of the particle  $P$ . Nevertheless, if we look at the measuring device, we certainly see that either  $\frac{\hbar}{2}$  or  $-\frac{\hbar}{2}$  spin component has been measured, that correspond to the states  $|m_\uparrow\rangle$  and  $|m_\downarrow\rangle$ , respectively. These are obviously not the same as the state (3). Indeed,  $|m_\uparrow\rangle$  and  $|m_\downarrow\rangle$  are pure states while  $\hat{\rho}_M$  is not. As is well known by now,  $\hat{\rho}_M$  cannot be considered a statistical mixture of  $|m_\uparrow\rangle$  and  $|m_\downarrow\rangle$ , i.e., "ignorance interpretation" cannot solve the problem of objectification[22]. Why do we get different states? According to orthodox quantum mechanics, one may argue as follows. The reduced density matrix  $\hat{\rho}_M$  has been calculated from the state  $|\Psi\rangle$  (cf. Eq.(2)) of the whole system  $P + M$ . A state is a result of a measurement (the preparation), so we may describe  $M$  by  $\hat{\rho}_M$  if we have gained our information about  $M$  from a measurement done on  $P + M$ . On the other hand, looking at the measuring device directly is equivalent with a measurement done directly on  $M$ . In this case  $M$  is described by either  $|m_\uparrow\rangle$  or  $|m_\downarrow\rangle$ . We may conclude that performing measurements on different systems (each containing the system we want to describe) gives rise to different descriptions (in terms of different states). Let us call the system which has been measured (it is  $P + M$  in the first case and  $M$  in the second case) the *quantum reference system*. Using this terminology, we may tell that we make a measurement on the quantum reference system  $R$ , thus we prepare its state  $|\psi_R\rangle$  and using this information we calculate the state  $\hat{\rho}_S(R) = Tr_{R\setminus S} |\psi_R\rangle\langle\psi_R|$  of a subsystem  $S$ . We shall call  $\hat{\rho}_S(R)$  the state of  $S$  with respect to  $R$ . Obviously  $\hat{\rho}_R(R) = |\psi_R\rangle\langle\psi_R|$ , thus  $|\psi_R\rangle$  may be identified with the state of the system  $R$  with respect to itself.

Let us emphasize that up to now, despite of the new terminology, there is nothing new in the discussion. We have merely considered some rather elementary consequences of basic quantum mechanics.

Let us return now to the question why the state of the system  $S$  (i.e.,  $\hat{\rho}_S(R)$ ) depends on

the choice of the quantum reference system  $R$ . In the spirit of the Copenhagen interpretation one would answer that in quantum mechanics measurements unavoidably disturb the systems, therefore, if we perform measurements on different surroundings  $R$ , this disturbance is different, and this is reflected in the  $R$ -dependence of  $\hat{\rho}_S(R)$ . Nevertheless, this argument is not compelling. We may also take the realist point of view and assume that the states of the systems have already existed before the measurements, and appropriate measurements do not change these states. Then the  $R$ -dependence of  $\hat{\rho}_S(R)$  becomes an inherent property of quantum mechanics, the states themselves represent actually existing properties (or correspond to some elements of the reality), and quantum formalism becomes a description of the reality rather than a calculational tool which relates consecutive measurement results. Let us leave at this decisive point the traditional framework of quantum mechanics and follow the new line just sketched. Below a comparison between the two approaches is given.

<b>Standard QM:</b>	<b>Present approach:</b>
Quantum reference system dependence is caused by the influence of the measurement. Measurement is a primary concept.	Quantum reference system dependence is a fundamental property. States exist and depend on quantum reference systems even in the absence of any measurement. Measurement is a derived concept.

The meaning of the quantum reference systems is now analogous to the classical coordinate systems. Choosing a classical coordinate system means that we imagine what we would experience if we were there. Similarly, choosing a quantum reference system  $R$  means that we imagine what we would experience if we performed a measurement on  $R$  that does not disturb  $\hat{\rho}_R(R) = |\psi_R\rangle\langle\psi_R|$ . In order to see that such a measurement exists, consider an operator  $\hat{A}$  (which acts on the Hilbert space of  $R$ ) whose eigenstates include  $|\psi_R\rangle$ . The measurement of  $\hat{A}$  will not disturb  $|\psi_R\rangle$ . Let us emphasize that the possibility of nondisturbing measurements is an expression of realism: the state  $\hat{\rho}_R(R)$  exists independently whether we

measure it or not.

Despite the analogy, there are several differences between the concept of classical and quantum reference systems. These are summarized below:

<b>Classical reference system (CRS):</b>	<b>Quantum reference system (QRS):</b>
CRS is an abstraction of actual physical systems, as their detailed structure is not important.	QRS is a physical system. The detailed structure cannot be eliminated.
measurements are done on a system by devices attached to the CRS	measurement is done on the QRS
there is a one-to-one relationship (transformation) between descriptions with respect to two different CRS-s.	there is a stochastic relationship between descriptions with respect to two different QRS-s (indeterminism), or they may not even be compared

As the dependence of  $\hat{\rho}_S(R)$  on  $R$  is a fundamental property now, one has to specify the relation of the different states and has to relate the formalism with experience. This can be done in terms of suitable postulates[13]. Below these postulates are listed and are applied to the theoretical description of measurements.

### 3 Rules of the new framework

**Postulate 1.** *The system  $S$  to be described is a subsystem of the reference system  $R$ .*

**Postulate 2.** *The state  $\hat{\rho}_S(R)$  is a positive definite, Hermitian operator with unit trace, acting on the Hilbert space of  $S$ .*

**Definition 1.**  *$\hat{\rho}_S(S)$  is called the internal state of  $S$ .*

**Postulate 3.** *The internal states  $\hat{\rho}_S(S)$  are always projectors, i.e.,  $\hat{\rho}_S(S) = |\psi_S\rangle\langle\psi_S|$ .*

In the following these projectors will be identified with the corresponding wave functions  $|\psi_S\rangle$  (as they are uniquely related, apart from a phase factor).

**Postulate 4.** *The state of a system  $S$  with respect to the reference system  $R$  (denoted by  $\hat{\rho}_S(R)$ ) is the reduced density matrix of  $S$  calculated from the internal state of  $R$ , i.e.  $\hat{\rho}_S(R) = Tr_{R \setminus S}(\hat{\rho}_R(R))$ , where  $R \setminus S$  stands for the subsystem of  $R$  complement to  $S$ .*

**Definition 2.** *An isolated system is such a system that has not been interacting with the outside world. A closed system is such a system that is not interacting with any other system at the given instant of time (but might have interacted in the past).*

**Postulate 5.** *If  $I$  is an isolated system then its state is independent of the reference system  $R$ :  $\hat{\rho}_I(R) = \hat{\rho}_I(I)$ .*

**Postulate 6.** *If the reference system  $R = I$  is an isolated one then the state  $\hat{\rho}_S(I)$  commutes with the internal state  $\hat{\rho}_S(S)$ .*

This means that the internal state of  $S$  coincides with one of the eigenstates of  $\hat{\rho}_S(I)$ .

**Definition 3.** *The possible internal states are the eigenstates of  $\hat{\rho}_S(I)$  provided that the reference system  $I$  is an isolated one.*

**Postulate 7.** *If  $I$  is an isolated system, then the probability  $P(S, j)$  that the eigenstate  $|\phi_{S,j}\rangle$  of  $\hat{\rho}_S(I)$  coincides with  $\hat{\rho}_S(S)$  is given by the corresponding eigenvalue  $\lambda_j$ .*

**Postulate 8.** *The result of a measurement is contained unambiguously in the internal state of the measuring device.*

**Postulate 9.** *If there are  $n$  ( $n = 2, 3, \dots$ ) disjointed physical systems, denoted by  $S_1, S_2, \dots, S_n$ , all contained in the isolated reference system  $I$  and having the possible internal states  $|\phi_{S_1,j}\rangle, |\phi_{S_2,j}\rangle, \dots, |\phi_{S_n,j}\rangle$ , respectively, then the joint probability that  $|\phi_{S_i,j_i}\rangle$  coincides with the internal state of  $S_i$  ( $i = 1, \dots, n$ ) is given by*

$$\begin{aligned} P(S_1, j_1, S_2, j_2, \dots, S_n, j_n) \\ = Tr_{S_1+S_2+\dots+S_n}[\hat{\pi}_{S_1,j_1} \hat{\pi}_{S_2,j_2} \dots \hat{\pi}_{S_n,j_n} \hat{\rho}_{S_1+S_2+\dots+S_n}(I)], \end{aligned} \quad (4)$$

where  $\hat{\pi}_{S_i,j_i} = |\phi_{S_i,j_i}\rangle\langle\phi_{S_i,j_i}|$ .

**Postulate 10.** *The internal state  $|\psi_C\rangle$  of a closed system  $C$  satisfies the time dependent Schrödinger equation  $i\hbar\partial_t|\psi_C\rangle = \hat{H}|\psi_C\rangle$ .*

Here  $\hat{H}$  stands for the Hamiltonian.

Let us emphasize that **Postulate 6** and **8** do not have any counterpart in standard quantum mechanics. They replace the measurement postulate but are not simple translations of that (as it will be shown below). The relation to the experience (that provides the whole construction with a physical meaning) is expressed by **Postulate 8**. On the other hand, **Postulate 6** and **Postulate 8** are mathematically equivalent with the proposals in Refs.[19],[18]. The present approach differs from these at the level of interpretation.

In order to demonstrate the working of the postulates let us consider again the measurement discussed in the previous section. A measurement is treated as a usual interaction between two systems, therefore, it is specified by a Hamiltonian or the corresponding unitary time evolution. For simplicity we assume that it is given by Eq.(2) and that the measuring device + measured object composite is an isolated system. Then **Postulate 5** and **Postulate 3** imply that  $\hat{\rho}_{P+M}(P + M) = |\Psi \rangle \langle \Psi|$ . According to **Postulate 4** the state of the measuring device with respect to the compound system  $P + M$  is

$$\hat{\rho}_M(P + M) = |m_{\uparrow} \rangle \langle \alpha|^2 \langle m_{\uparrow}| + |m_{\downarrow} \rangle \langle \beta|^2 \langle m_{\downarrow}| \quad . \quad (5)$$

Applying **Postulate 6** we get that the internal state  $\hat{\rho}_M(M)$  (cf. **Definition 1**) of the measuring device is either  $|m_{\uparrow} \rangle \langle m_{\uparrow}|$  (with probability  $|\alpha|^2$ , according to **Postulate 7**) or  $|m_{\downarrow} \rangle \langle m_{\downarrow}|$  (with probability  $|\beta|^2$ ). Finally, **Postulate 8** tells us that the actual measurement results (the experience) correspond to this  $\hat{\rho}_M(M)$ . As one can see, we get the same result as in orthodox quantum mechanics (this time without assuming the collapse of the wave function). One reason for this was the special dynamics (2). In orthodox quantum mechanics one usually specifies only which observable has been measured. It tacitly assumes a simple approximation for the dynamics of the measurement like (2). In such cases our postulates lead to the same results. If the dynamics is different (e.g., a more detailed description is given), nothing ensures that one gets back the results of the traditional approach exactly. This means that the above postulates cannot be considered as mere translations of orthodox quantum mechanics. One should also be aware that the present approach leaves much less flexibility than orthodox quantum mechanics. While in the traditional approach one may rather freely choose

a bordering line between quantum and classical regime, according to the present approach such bordering line does not exist, and one should in principle always apply a quantum description. Based on this description one may justify the approximations which lead to the traditional approach.

## 4 Non-comparability of the states

It is an important feature of the present approach that the states defined with respect to different quantum reference systems are not necessarily comparable. Below we explain what it means.

The present approach works as follows. We assume that the internal state of an isolated system  $I$  is known. Then the states of all the subsystems  $S_j$  of  $I$  with respect to  $I$  (i.e.,  $\hat{\rho}_{S_j}(I)$ ) are also known together with their possible internal states (i.e., the eigenstates of  $\hat{\rho}_{S_j}(I)$ ). The aim of a measurement performed on  $S_j$  is to learn which of the possible internal states is the actual one. This can be done without disturbing that state. It can be achieved if an operator commuting with  $\hat{\rho}_{S_j}(I)$  is measured, i.e., if the dynamics of the measurement is approximately given by

$$|\phi_{S_j,k} \rangle |m_0 \rangle \rightarrow |\phi_{S_j,k} \rangle |m_k \rangle \quad , \quad (6)$$

where  $|\phi_{S_j,k} \rangle$  stands for the  $k$ -th possible internal state of  $S_j$  (cf. **Definition 3**).

If two subsystems  $S_1$  and  $S_2$  are disjoint ( $S_1 \cap S_2 = \emptyset$ ) then one can perform such nondisturbing measurements on both systems simultaneously without disturbing either  $\hat{\rho}_{S_1}(I)$  or  $\hat{\rho}_{S_2}(I)$ . Correspondingly, **Postulate 9** provides us with a positive definite expression for the joint probability that these states coincide with specified possible internal states.

There is a special situation if  $S_2 = I \setminus S_1$ . The internal state of the isolated system can be expressed in this case by the possible internal states of  $S_1$  and  $S_2$  as

$$|\psi_I \rangle = \sum_k c_k |\phi_{S_1,k} \rangle |\phi_{S_2,k} \rangle \quad . \quad (7)$$

This is the Schmidt representation[17]. It readily implies that there is a unique relationship between the internal state of  $S_1$  and that of  $S_2$ . Indeed, Eq.(4) yields

$$P(S_1, j_1, S_2, j_2) = P(S_1, j_1) \delta_{j_1 j_2} \quad (8)$$

or

$$P(S_2, j_2 | S_1, j_1) = \delta_{j_1 j_2} . \quad (9)$$

We can rewrite Eq.(4) in the general case as follows:

$$P(S_1, j_1, \dots, S_n, j_n) = \langle \psi_I | \phi_{S_1, j_1} \rangle \langle \phi_{S_1, j_1} | \dots | \phi_{S_n, j_n} \rangle \langle \phi_{S_1, j_1} | \psi_I \rangle . \quad (10)$$

Eq.(7) implies

$$\langle \psi_I | \phi_{S_1, j_1} \rangle \langle \phi_{S_1, j_1} | = \sum_k c_k^* \langle \phi_{I \setminus S_1, k} | \langle \phi_{S_1, k} | \phi_{S_1, j_1} \rangle \langle \phi_{S_1, j_1} | \quad (11)$$

$$= c_{j_1}^* \langle \phi_{S_1, j_1} | \langle \phi_{I \setminus S_1, k} | = \langle \psi_I | \phi_{I \setminus S_1, j_1} \rangle \langle \phi_{I \setminus S_1, j_1} | \quad (12)$$

i.e., one can replace  $\hat{\pi}_{S_1, j_1} = |\phi_{S_1, j_1} \rangle \langle \phi_{S_1, j_1}|$  by  $\hat{\pi}_{I \setminus S_1, j_1} = |\phi_{I \setminus S_1, j_1} \rangle \langle \phi_{I \setminus S_1, j_1}|$  in Eq.(4). This sometimes makes possible simultaneous nondisturbing check of internal states of non-disjoint systems.

Suppose, e.g., that  $I = S_1 + S_2 + S_3$  (where  $S_i \neq \emptyset$  and  $S_i \cap S_j = \emptyset$  if  $i \neq j$ ) and we would like to learn the internal state of  $A = S_1 + S_2$  and of  $B = S_2 + S_3$ . The systems  $A$  and  $B$  are not disjoint, but their complementary systems  $S_3$  and  $S_1$  are. Therefore, one may perform nondisturbing measurements on  $S_3$  and  $S_1$ . The knowledge of the internal states of these systems uniquely specifies the internal states of the original systems  $A$  and  $B$  (cf. Eq.(9)). Correspondingly, Eq.(4) yields a positive definite expression for  $P(A, j, B, k)$ .

There are, however, situations when neither the systems nor their complementary systems are disjoint. The simplest case is if  $I = S_1 + S_2 + S_3 + S_4$  (where  $S_i \neq \emptyset$  and  $S_i \cap S_j = \emptyset$  if  $i \neq j$ ) and we would like to learn the internal state of  $A = S_1 + S_2$  and of  $B = S_2 + S_3$ . Obviously,

$$A \neq B, A \cap B = S_2 \neq \emptyset \quad (13)$$

$$(I \setminus A) \cap B = (S_3 + S_4) \cap (S_2 + S_3) = S_3 \neq \emptyset \quad (14)$$

$$A \cap (I \setminus B) = (S_1 + S_2) \cap (S_1 + S_4) = S_1 \neq \emptyset \quad (15)$$

$$(I \setminus A) \cap (I \setminus B) = (S_3 + S_4) \cap (S_1 + S_4) = S_4 \neq \emptyset \quad (16)$$

Thus, one cannot relate this situation to the case of disjoint subsystems. Indeed, a measurement done on  $A$  or on  $I \setminus A$  (which does not disturb  $\hat{\rho}_A(A)$ ) usually does disturb  $\hat{\rho}_B(B)$  and  $\hat{\rho}_{I \setminus B}(I \setminus B)$ . Therefore, there is no way to perform a measurement which conveys information about both the internal state of  $A$  and that of  $B$ . This is reflected mathematically by the postulates, namely, the positivity of  $P(A, i, B, k)$  (if Eq.(4) is formally applied) is not guaranteed (it is usually not even real). In such a situation the states  $\hat{\rho}_A(A)$ ,  $\hat{\rho}_B(B)$  cannot be compared, although both exist separately. Conclusions based on assumptions about the simultaneous existence of  $\hat{\rho}_A(A)$  and  $\hat{\rho}_B(B)$  (in the sense that one imagines that he knows both states) or, especially, about a related joint probability can lead to contradictions. Therefore, non-comparability of existing states is an essential feature of the present approach.

## 5 Bell's inequality and the principle of locality

Let us consider a Bell-type experiment performed on two spin-1/2 particles. In order to exhibit the mathematical structure we write the internal state of the two particle system before the measurements as

$$\sum_j c_j |\phi_{P_1, j} \rangle |\phi_{P_2, j} \rangle \quad (17)$$

where  $c_1 = a$ ,  $c_2 = -b$ ,  $|\phi_{P_1, 1} \rangle = |1, \uparrow \rangle$ ,  $|\phi_{P_1, 2} \rangle = |1, \downarrow \rangle$ ,  $|\phi_{P_2, 1} \rangle = |2, \downarrow \rangle$ ,  $|\phi_{P_2, 2} \rangle = |2, \uparrow \rangle$ . Eq.(17) is just the Schmidt representation, thus  $P(P_1, j, P_2, k) = |c_j|^2 \delta_{j, k}$ .

Let us consider now a typical experimental situation, when measurements on both particles are performed. We shall show that according to the present theory the observed correlations are exclusively due to the previous interaction between the particles. Before the measurements the internal state of the isolated system  $P_1 + M_1 + P_2 + M_2$  ( $P_1, P_2$  stands for the particles and  $M_1, M_2$  for the measuring devices, respectively) is given by

$(\sum_j c_j |\phi_{P_1,j} > |\phi_{P_2,j} >) |m_0^{(1)} > |m_0^{(2)} >$ , while it is

$$\sum_j c_j \hat{U}_t(P_1 + M_1) (|\phi_{P_1,j} > |m_0^{(1)} >) \hat{U}_t(P_2 + M_2) (|\phi_{P_2,j} > |m_0^{(2)} >) \quad , \quad (18)$$

a time  $t$  later, i.e. during and after the measurements. Here  $\hat{U}_t(P_i + M_i)$  ( $i = 1, 2$ ) stands for the unitary time evolution operator of the closed system  $P_i + M_i$ .

Eq.(18) implies that the internal states of the closed systems  $P_1 + M_1$  and  $P_2 + M_2$  evolve unitarily and do not influence each other. This follows readily if one applies **Postulates 4, 6** to Eq.(18) and takes into account the unitarity of  $\hat{U}_t(P_i + M_i)$ . This time evolution can be given explicitly through the relations

$$|\xi(P_i, j) > |m_0^{(i)} > \rightarrow |\xi(P_i, j) > |m_j^{(i)} > \quad , \quad (19)$$

where  $i, j = 1, 2$  and  $|\xi(P_i, j) >$  is the  $j$ -th eigenstate of the spin measured on the  $i$ -th particle along an axis  $z^{(i)}$  which closes an angle  $\vartheta_i$  with the original  $z$  direction. The time evolution of the internal state of the closed systems  $P_i + M_i$  is given explicitly by  $|\psi_{P_i} > |m_0^{(i)} > \rightarrow \sum_j \langle \phi_{P_i,j} | \psi_{P_i} > |\phi_{P_i,j} > |m_j^{(i)} >$ . As we see, the  $i$ -th measurement process is completely determined by the initial internal states of the particle  $P_i$ . Therefore, any correlation between the measurements may only stem from the initial correlation of the internal states of the particles.

For the calculation of the state  $\hat{\rho}_{M_1}(M_1)$  (which corresponds to the measured value, cf. **Postulate 8**) one needs to know the state of the whole isolated system  $P_1 + P_2 + M_1 + M_2$ . Using Eq.(19) the final state (18) may be written as

$$\begin{aligned} \sum_{j,k} \left( \sum_l c_l \langle \xi(P_1, j) | \phi_{P_1,l} > \langle \xi(P_2, k) | \phi_{P_2,l} > \right) \\ \times |m_j^{(1)} > |m_k^{(2)} > |\xi(P_1, j) > |\xi(P_2, k) > . \end{aligned}$$

Direct calculation shows that

$$\begin{aligned} \hat{\rho}_{M_1}(P_1 + P_2 + M_1 + M_2) \\ = \sum_j \left( \sum_l |c_l|^2 \langle \xi(P_1, j) | \phi_{P_1,l} > |^2 \right) |m_j^{(1)} > \langle m_j^{(1)}|. \end{aligned}$$

Note that it is independent of the second measurement.

According to **Postulate 6**  $|\psi_{M_1}\rangle$  is one of the  $|m_j^{(1)}\rangle$ -s. (Similarly one may derive that  $|\psi_{M_2}\rangle$  is one of the  $|m_k^{(2)}\rangle$ -s.) The probability of the observation of the  $j$ -th result (up or down spin in a chosen direction) is

$$P(M_1, j) = \sum_l |c_l|^2 < \xi(P_1, j) | \phi_{P_1, l} \rangle|^2. \quad (20)$$

This may be interpreted in conventional terms:  $|c_l|^2$  is the probability that  $|\psi_{P_1}\rangle = |\phi_{P_1, l}\rangle$ , and  $| < \xi(P_1, j) | \phi_{P_1, l} \rangle|^2$  is the conditional probability that one gets the  $j$ -th result if  $|\psi_{P_1}\rangle = |\phi_{P_1, l}\rangle$ . Indeed, the initial internal state of  $P_1$  uniquely determines the internal state of  $P_1 + M_1$  (owing to the unitary time evolution), and the internal state of  $P_1 + M_1$  uniquely determines the internal state of its complementary system  $P_2 + M_2$  (and vica versa). Therefore, the joint probability that the initial internal state of  $P_1$  coincides with its  $l$ -th possible internal state and the final internal state of  $M_1$  coincides with its  $j$ -th possible internal state is the same as the joint probability  $P(P_2 + M_2, l, M_1, j)$  that the final internal state of  $P_2 + M_2$  coincides with its  $l$ -th possible internal state and the final internal state of  $M_1$  coincides with its  $j$ -th possible internal state. As  $P_2 + M_2$  and  $M_1$  are disjoint systems,  $P(P_2 + M_2, l, M_1, j)$  is positive definite, namely, if calculated according to Eq.(18), it is just the summand on the right hand side of Eq.(20).

Thus we see that the initial internal state of  $P_1$  determines the outcome of the first measurement in the usual probabilistic sense. One may show quite similarly that the initial internal state of  $P_2$  determines the outcome of the second measurement in the same way. In this sense the internal states of  $P_1$  and  $P_2$  play the role of local hidden variables. On the other hand, to hidden variables some supernatural features are attributed (e.g. that they cannot be measured in any way) and they are assumed to be comparable with the results of both measurements. None of these properties applies to the internal states of  $P_1$  and  $P_2$ . As for the non-comparability we mean that the initial internal state of  $P_1$  and  $P_2$  is not comparable with both  $|\psi_{M_1}\rangle$  and  $|\psi_{M_2}\rangle$  (while they can be compared with one of them). This means that in our theory there is no way to define the joint probability  $P(P_1, l_1, P_2, l_2, (0); M_1, j, M_2, k, (t))$ ,

i.e., the probability that initially  $|\psi_{P_1}\rangle = |\phi_{P_1,l_1}\rangle$  and  $|\psi_{P_2}\rangle = |\phi_{P_2,l_2}\rangle$  and finally  $|\psi_{M_1}\rangle = |m_j^{(1)}\rangle$  and  $|\psi_{M_2}\rangle = |m_k^{(2)}\rangle$ . Intuitively we would write

$$\begin{aligned} P(P_1, l_1, P_2, l_2, (0); M_1, j, M_2, k, (t)) \\ = |c_{l_1}|^2 \delta_{l_1, l_2} | \langle \xi(P_1, j) | \phi_{P_1, l_1} \rangle |^2 | \langle \xi(P_2, k) | \phi_{P_2, l_2} \rangle |^2, \end{aligned} \quad (21)$$

as  $|c_{l_1}|^2 \delta_{l_1, l_2}$  is the joint probability that  $|\psi_{P_1}\rangle = |\phi_{P_1, l_1}\rangle$  and  $|\psi_{P_2}\rangle = |\phi_{P_2, l_2}\rangle$ , and  $| \langle \xi(P_i, j) | \phi_{P_i, l_i} \rangle |^2$  is the conditional probability that one gets the  $j$ -th result in the  $i$ -th measurement if initially  $|\psi_{P_i}\rangle = |\phi_{P_i, l_i}\rangle$  ( $i = 1, 2$ ). Certainly the existence of such a joint probability would immediately imply the validity of Bell's inequality, thus it is absolutely important to understand why this probability does not exist.

As our postulates provide us with equal time joint probabilities, we should try to express  $P(P_1, l_1, P_2, l_2, (0); M_1, j, M_2, k, (t))$  with them. As above, we may note that the initial internal state of  $P_1$  is uniquely related to the final internal state of  $P_1 + M_1$ , the initial internal state of  $P_2$  is uniquely related to the final internal state of  $P_2 + M_2$ , and the final internal state of  $P_1 + M_1$  is uniquely related to the final internal state of  $P_2 + M_2$ . Therefore, if  $P(P_1, l_1, P_2, l_2, (0); M_1, j, M_2, k, (t))$  exists, it coincides with  $P(P_1 + M_1, l_1, M_1, j, M_2, k) \delta_{l_1, l_2}$ . But the systems  $P_1 + M_2$ ,  $M_1$  and  $M_2$  are not disjoint, neither are their complementary systems. As a result, if one tries to apply Eq.(4), one obtains

$$\begin{aligned} P(P_1 + M_1, l_1, M_1, j, M_2, k) &= \sum_{j'} \langle \xi(P_1, j') | \phi_{P_1, l_1} \rangle \langle \phi_{P_1, l_1} | \xi(P_1, j) \rangle \\ &\quad \times \left( \sum_{l'} c_{l'} \langle \xi(P_1, j) | \phi_{P_1, l'} \rangle \langle \xi(P_2, k) | \phi_{P_2, l'} \rangle \right) \\ &\quad \times \left( \sum_{l''} c_{l''}^* \langle \xi(P_1, j') | \phi_{P_1, l''} \rangle^* \langle \xi(P_2, k) | \phi_{P_2, l''} \rangle^* \right) \end{aligned} \quad (22)$$

This expression fails to be real and positive when Bell's inequality is violated. This can be seen because summing Eq.(22) over  $l_1$  one gets the correct joint probability

$$P(M_1, j, M_2, k) = \left| \sum_l c_l \langle \xi(P_1, j) | \phi_{P_1, l} \rangle \langle \xi(P_2, k) | \phi_{P_2, l} \rangle \right|^2. \quad (23)$$

which can also be obtained directly from Eq.(4). This is the usual quantum mechanical expression which violates Bell's inequality and whose correctness is experimentally proven.

The other side of the non-comparability is that a nondisturbing measurement of  $|\psi_{P_1+M_1} \rangle$  inevitably disturbs  $|\psi_{P_1+M_1+P_2+M_2} \rangle$ , and thus  $P(M_1, j, M_2, k)$ , too. Therefore, if one measures  $|\psi_{P_1+M_1} \rangle$ ,  $|\psi_{M_1}$  and  $|\psi_{M_2}$  (in order to give an operational definition for  $P(P_1 + M_1, l_1, M_1, j, M_2, k)$ )  $P(M_1, j, M_2, k)$  will be changed. Indeed, after having measured  $|\psi_{P_1+M_1} \rangle$ <sup>2</sup> by a further measuring device  $M_3$  we get for the internal state of the whole system

$$\sum_l c_l \left( \sum_j \langle \xi(P_1, j) | \phi_{P_1, l} \rangle | \xi(P_1, j) \rangle | m_j^{(1)} \rangle \right) \times \left( \sum_k \langle \xi(P_2, k) | \phi_{P_2, l} \rangle | \xi(P_2, k) \rangle | m_k^{(2)} \rangle \right) | m_l^{(3)} \rangle . \quad (24)$$

As the systems  $M_1$ ,  $M_2$ ,  $M_3$  are disjointed, we may apply **Postulate 9** for  $n = 3$  and we get for  $P(M_3, l_1, M_1, j, M_2, k)$  the positive definite expression

$$P(M_3, l_1, M_1, j, M_2, k) = |c_{l_1}|^2 | \langle \xi(P_1, j) | \phi_{P_1, l_1} \rangle |^2 | \langle \xi(P_2, k) | \phi_{P_2, l_1} \rangle |^2 . \quad (25)$$

This readily implies Bell's inequality. Indeed, if **Postulate 9** is applied for Eq.(24) one gets

$$P(M_1, j, M_2, k) = \sum_l |c_l|^2 | \langle \xi(P_1, j) | \phi_{P_1, l} \rangle |^2 | \langle \xi(P_2, k) | \phi_{P_2, l} \rangle |^2 \quad (26)$$

which satisfies Bell's inequality and differs from Eq.(23). This is due to the extra measurement which is equivalent with a measurement of the "hidden variable".

Summarizing, we have seen that the initial internal state of  $P_1$  ( $P_2$ ) determines the first (second) measurement process, therefore, these states 'carry' the initial correlations and 'transfer' them to the measuring devices. As the measurement processes do not influence each other, the observed correlations may stem only from the 'common past' of the particles. On the other hand, any attempt to compare the initial internal states of  $P_1$  and  $P_2$  with the results of both measurements changes the correlations, thus a joint probability for the simultaneous existence of these states cannot be defined. This means that the reason for the violation of Bell's inequality is that the usual derivations always assume that the states (or "stable properties", "hidden variables" etc.) which carry the initial correlations can be freely

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<sup>2</sup> This is equivalent by recording the initial internal state of  $P_1$ .

compared with the results of the measurements. This comparability is usually thought to be a consequence of realism. According to the present theory, the above assumption goes beyond the requirements of realism and proves to be wrong, because each of the states  $|\psi_{P_1+M_1}\rangle$ ,  $|\psi_{P_2+M_2}\rangle$ ,  $|\psi_{M_1}\rangle$  and  $|\psi_{M_2}\rangle$  exists individually, but they cannot be compared without changing the correlations.

## 6 Summary and conclusion

A new approach to quantum mechanics was presented. The measurement postulates were abandoned and replaced by different ones to get a self-consistent, universally valid quantum mechanics. The present approach was based on the idea of quantum reference systems and the relational nature of quantum states, and, from the mathematical point of view the Schmidt representation (eigenstates of density matrices) was utilized. It was briefly shown how this idea solved the objectification problem, then the concept of non-comparability was discussed. Finally, it was shown that locality was maintained in a Bell-type experiment and the violation of Bell's inequality was rendered possible by non-comparability of existing states.

According to the present approach the principle of locality is valid in quantum mechanics, and it is the concept of realism that should be modified. This modification amounts to surrendering the overall comparability of existing things (states) which are defined with respect to different (quantum) reference systems.

Finally, let us mention that the present approach is self-contained, i.e., it does not rely upon the concept of *a priori* classical objects. This means that (if correct) it should give account of classical behavior. This is a great challenge but it has not been discussed here.

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